HW1 Solutions

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1 Notation

Throughout the solutions I will use the units where $c = 1$. Also, the abbreviations $S = \sinh \eta$, $C = \cosh \eta$, $T = \tanh \eta$ will be used.

2 Problem1

a).In an arbitrary frame S we have two events (t_1, x_1) and (t_2, x_2) such that $\Delta t = t_1 - t_2 > 0$. We wish to go to a frame S' where the order of the events is reversed (i.e. $\Delta t' = t'_1 - t'_2 < 0$). But

$$\Delta t' = t'_1 - t'_2 = \gamma((t_1 - t_2) - \beta(x_1 - x_2))$$

Since $\beta < 1$, Δx must be greater than Δt. Thus $(\Delta t)^2 - (\Delta x)^2 < 0$, which corresponds to a space-like separation.

b). We wish to go to a frame where $\Delta x' = 0$. Since

$$\Delta x' = \gamma(-\beta \Delta t + \Delta x),$$

this is possible as long as $\Delta t < \Delta x$ (i.e the separation is time-like). Since the proper time τ is defined by

$$\tau^2 = (\Delta t)^2 - (\Delta x)^2,$$

in the frame where $\Delta x = 0$, $\tau = \Delta t$.

3 Problem2

a). Just multiply Λ by (t, x).

b). Using the fact that $T = S/C$ and $C^2 - S^2 = 1$ we have

$$\gamma^2 = \frac{1}{1 - T^2} = \frac{C^2}{C^2 - S^2} = C^2$$

And

$$\gamma\beta = C\frac{S}{C} = S$$

c). Using the fact that C is even and S and T are odd functions we have

$$\Lambda(-\beta) = \Lambda(-\eta) = \begin{array}{cc} C & S \\ S & C \end{array} \tag{1}$$

Thus

$$\Lambda(-\beta)\Lambda(\beta) = \begin{array}{cc} C^2 - S^2 & -CS + SC \\ -CS + SC & C^2 - S^2 \end{array} \tag{2}$$

4 Problem3

Simultaneous events in one frame are not simultaneous in the other. Thus the runner should not be worried, since for him the front door will close before the back door.

5 Problem4

a). The acceleration at a given moment is specified with respect to the instantaneously comoving frame (i.e. the frame in which the astronaut is stationary at that moment).
   b). $\beta = \tanh(g\tau)$.
   d). In ten years in the astronaut's frame (i.e. $\tau = 10$) the ship will cover the distance of $7.5 * 10^6 ly$. Thus the engineer is correct.

6 Problem5

Given that $u = T(a)$, $v = T(b)$, $u' = T(a - b)$, using Eq.(2.41) we have

$$T(a - b) = u' = \frac{u - v}{1 - vu} = \frac{T(a) - T(b)}{1 - T(a)T(b)}$$

7 Problem6

First, I do the conservation of energy/momentum in the center of mass frame and then boost to the lab frame.

The center of mass frame is the frame where the two incoming protons have the same magnitude of momentum. Thus $\vec{p_i} = \vec{p_f} = 0$. Thus at threshold all 4 particles will be stationary and all of their energy will be mass energy. Therfore

$$E_f = 4m = E_i = 2\gamma m$$

Thus in the c.o.m. frame $2 = \gamma = \cosh \eta$.

In the lab frame one of the protons is stationary (i.e. its $\vec{p} = 0 = m \sinh(0) = m \sinh(-\eta + \eta_{boost})$). Thus to go to the lab frame we need to boost by $\eta_{boost} = \eta$, and the energy of the other proton will become

$$m \cosh(\eta + \eta_{boost}) = m \cosh(2\eta) = 0.94 * 10^9 (\cosh(2 \cosh^{-1} 2)) = 6.58 * 10^9.$$

8 Problem7

Since the pion is stationary when it decays $0 = \vec{p_i} = \vec{p_f} = \vec{p_\mu} + \vec{p_\nu}$, thus the pion and the muon have the same magnitude of momentum p. Using the fact that $E^2 - p^2 = m^2$ for any particle we have

$$E_i = m = E_f = E_\mu + E_\nu = \sqrt{(\frac{3}{4}m)^2 + p^2} + p.$$

Thus $p = \frac{7}{32}m = m\beta_0$.

9 Problem8

a).
$$E = 0.25 * 10^{12} = m \cosh \eta_f = 0.5 * 10^6 \cosh \eta_f$$

Thus $\eta_f = 13.8$.

b,c). To find the acceleration and proper time solve the equations

$$c\eta_f = g\tau_f$$

$$x = \frac{c^2}{g}[\cosh \frac{g\tau_f}{c} - 1]$$

given in problem 4. Then, $g = 4.1 * 10^{20} m/s$ and $\tau_f = 1.01 * 10^{-11}$.

To find the lab time integrate

$$dt = \cosh \frac{g\tau}{c} d\tau.$$

Then

$$t = \frac{c}{g} \sinh \frac{g\tau_f}{c} = 3.66 * 10^{-7},$$

and

$$\frac{t}{\tau} = \frac{\sinh(\eta_f)}{\eta_f} = 3.62 * 10^4.$$